Research Article

Mathematical Methods in the Applied Sciences

Received 10 December 2014 Published online 3 June 2015 in Wiley Online Library

(wileyonlinelibrary.com) DOI: 10.1002/mma.3539
MOS subject classification: 76N10 (35B40 35Q35)

Asymptotic behavior for the one-dimensional *p*th power Newtonian fluid in unbounded domains

Ling Wan and Tao Wang[*†]

Communicated by M. Renardy

We consider the initial and initial-boundary value problems for a one-dimensional *p*th power Newtonian fluid in unbounded domains with general large initial data. We show that the specific volume and the temperature are bounded from below and above uniformly in time and space and that the global solution is asymptotically stable as the time tends to infinity. Copyright © 2015 John Wiley & Sons, Ltd.

Keywords: *p*th power Newtonian fluid; large initial data; asymptotically stable; unbounded domains

1. Introduction

The one-dimensional motion of a *p*th power Newtonian fluid can be described by the Navier–Stokes system for compressible flow in Lagrangian coordinates:

$$\begin{cases} v_t - u_x = 0, \\ u_t + \mathcal{P}_x = \left(\mu \dfrac{u_x}{v}\right)_x, \\ e_t + \mathcal{P} u_x = \dfrac{\mu u_x^2}{v} + \left(\kappa \dfrac{\theta_x}{v}\right)_x, \end{cases} \tag{1.1}$$

where $t > 0$ is the time variable, $x \in \Omega \subset \mathbb{R}$ is the Lagrangian space variable, and the primary dependent variables are the specific volume v, fluid velocity u, and temperature θ. The pressure $\mathcal{P}$ and the specific internal energy e are given by

$$\mathcal{P} = \frac{\theta}{v^p}, \quad e = c_v \theta \tag{1.2}$$

with the pressure exponent $p \geq 1$ and constant-specific heat $c_v > 0$. The viscosity coefficient μ and the heat conductivity κ are assumed to be positive constants.

The systems (1.1) and (1.2) are supplemented with the initial conditions

$$(v, u, \theta)|_{t=0} = (v_0, u_0, \theta_0) \qquad \text{on } \overline{\Omega}, \tag{1.3}$$

and one type of the following far-field and boundary conditions:

$$\lim_{x \to \pm\infty} (v_0(x), u_0(x), \theta_0(x)) = (1, 0, 1), \qquad \text{if } \Omega = \mathbb{R}; \tag{1.4}$$

$$(u, \theta_x)|_{x=0} = 0, \ \lim_{x \to \infty} (v_0(x), u_0(x), \theta_0(x)) = (1, 0, 1), \qquad \text{if } \Omega = (0, \infty); \tag{1.5}$$

School of Mathematics and Statistics, Wuhan University, Wuhan 430072, China
** Correspondence to: Tao Wang, School of Mathematics and Statistics, Wuhan University, Wuhan 430072, China.*
† *E-mail: tao.wang@whu.edu.cn*

$$(u,\theta)|_{x=0}=(0,1),\ \lim_{x\to\infty}(v_0(x),u_0(x),\theta_0(x))=(1,0,1),\qquad \text{if } \Omega=(0,\infty). \tag{1.6}$$

The aim of this article is to show the large-time behavior of solutions to the initial value problems (1.1)–(1.4) and the initial-boundary value problems (1.1)–(1.3) and (1.5), and (1.1)–(1.3) and (1.6) for general large initial data.

Now let us first recall some previous results in this direction. Lewicka and Watson [1] showed exponential convergence of solutions to equilibria for initial-boundary value problems involving fixed endpoints held at a fixed temperature or insulated. Qin and Huang [2] proved the regularity and exponential stability of solutions in $H^i (i=2,4)$ for (1.1) and (1.2) in bounded domains. Recently, Cui and Yao [3] established the large-time behavior of the global spherically or cylindrically symmetric solutions in H^1 for the pth power Newtonian fluid in multi-dimension.

We note that the papers [1–3] are all concerned with the asymptotic behavior of solutions for the pth power Newtonian fluid in bounded domains. Thus, it is natural to investigate the large-time behavior of solutions to (1.1) and (1.2) in unbounded domains. Our study is motivated by the previous works [4–6] on the large-time behavior of solutions for the ideal polytropic gases (1.1) and (1.2) with $p=1$.

Our first result is as follows.

Theorem 1
If the initial data (v_0,u_0,θ_0) satisfy

$$(v_0-1,u_0,\theta_0-1)\in H^1(\Omega),\quad \inf_{x\in\Omega} v_0(x)>0,\quad \inf_{x\in\Omega}\theta_0(x)>0, \tag{1.7}$$

and are compatible with (1.5) or (1.6) when $\Omega=(0,\infty)$, then there exists a unique global solution (v,u,θ) with positive θ to the problems (1.1)–(1.4) or (1.1)–(1.3) and (1.5), or (1.1)–(1.3) and (1.6) such that for each $T>0$,

$$\begin{gathered}(v-1,u,\theta-1)\in L^\infty\left(0,T;H^1(\Omega)\right),\quad v_t\in L^\infty\left(0,T;L^2(\Omega)\right),\\ (u_t,\theta_t,v_{xt},u_{xx},\theta_{xx})\in L^2\left(0,T;L^2(\Omega)\right).\end{gathered} \tag{1.8}$$

Moreover, there exists a positive constant C_0, depending only on $\|(v_0-1,u_0,\theta_0-1)\|_{H^1(\Omega)}$, $\inf_{x\in\Omega} v_0(x)$, and $\inf_{x\in\Omega}\theta_0(x)$, such that

$$C_0^{-1}\le v(t,x)\le C_0\quad \text{for all } (t,x)\in[0,\infty)\times\overline{\Omega}. \tag{1.9}$$

By the same calculations as those in [6], we can prove Theorem 2.

Theorem 2
Let (v_0,u_0,θ_0) satisfy (1.7) and be compatible with (1.5) or (1.6) when $\Omega=(0,\infty)$, and let (v,u,θ) be the (unique) solution to (1.1)–(1.4) or (1.1)–(1.3) and (1.5), or (1.1)–(1.3) and (1.6). Then there exists a positive constant C_1, depending solely on $\|(v_0-1,u_0,\theta_0-1)\|_{H^1(\Omega)}$, $\inf_{x\in\Omega} v_0(x)$ and $\inf_{x\in\Omega}\theta_0(x)$, such that

$$C_1^{-1}\le \theta(t,x)\le C_1\quad \text{for all } (t,x)\in[0,\infty)\times\overline{\Omega}, \tag{1.10}$$

$$\sup_{0\le t<\infty}\|(v-1,u,\theta-1)(t)\|^2_{H^1(\Omega)}+\int_0^\infty\left[\|v_y(t)\|^2_{L^2(\Omega)}+\|(u_x,\theta_x)(t)\|^2_{H^1(\Omega)}\right]\mathrm{d}t\le C_1, \tag{1.11}$$

$$\lim_{t\to\infty}\left(\|(v-1,u,\theta-1)(t)\|_{L^p(\Omega)}+\|(v_x,u_x,\theta_x)(t)\|_{L^2(\Omega)}\right)=0 \tag{1.12}$$

for each $p\in(2,\infty]$.

We omit the proof, because it is almost identical to that in [6] with the help of (1.9).

Remark 1.1
Theorems 1 and 2 generalize the results of [4–6] that are concerned with the large-time behavior of solutions for the case $p=1$. Theorem 2 also extends the results of [1] where the authors investigated systems (1.1) and (1.2) in bounded domains.

2. Proof of Theorem 1

This section is devoted to proving Theorem 1. We first deduce certain a priori estimates on the solutions $(v(t,x),u(t,x),\theta(t,x))$ for $x\in\Omega$ and $t\in[0,T]$ with T being a positive fixed constant. Letter C will be employed to denote the generic positive constant, which is independent of T and may vary from line to line. We will use $A\lesssim B$ ($B\gtrsim A$) if $A\le CB$ for some positive constant C. The notation $A\sim B$ means that $A\lesssim B$ and $B\lesssim A$.

We begin with the fundamental entropy-type energy estimate for the general exponent $p\ge 1$.

Lemma 2.1
The following estimate holds:

$$\sup_{0\le t\le T}\int_\Omega \eta(v,u,\theta)\mathrm{d}x+\int_0^T\int_\Omega\left[\frac{\mu u_x^2}{v\theta}+\frac{\kappa\theta_x^2}{v\theta^2}\right]\mathrm{d}x\mathrm{d}t\lesssim 1, \tag{2.1}$$

where $\eta(v,u,\theta)=\psi(v)+\frac{1}{2}u^2+c_v\phi(\theta)$ with $\phi(z)=z-\ln z-1$ and

$$\psi(z)=\begin{cases}\phi(z), & p=1,\\ z-\frac{1}{p-1}\left(1-z^{1-p}\right)-1, & p>1.\end{cases} \tag{2.2}$$

Proof
We have from (1.1) that the temperature θ satisfies

$$c_v\theta_t+\frac{\theta u_x}{v^p}=\left(\frac{\kappa\theta_x}{v}\right)_x+\frac{\mu u_x^2}{v}. \tag{2.3}$$

By multiplying $(1.1)_1$ by $(1-v^{-p})$, $(1.1)_2$ by u, and (2.3) by $(1-\theta^{-1})$, we conclude

$$\eta(v,u,\theta)_t+\frac{\mu u_x^2}{v\theta}+\frac{\kappa\theta_x^2}{v\theta^2}=\left[\frac{\mu uu_x}{v}+\left(1-\frac{1}{\theta}\right)\frac{\kappa\theta_x}{v}-u\left(\frac{\theta}{v^p}-1\right)\right]_x.$$

We integrate this last identity over $[0,T]\times\Omega$ to deduce (2.1) by recalling the far-field and boundary conditions (1.4)–(1.6). □

If we follow the argument in [7,8] and apply the Jensen inequality to convex functions ϕ and ψ, we can deduce the following lemma from the estimate (2.1).

Lemma 2.2
For all integer i and $t\in[0,T]$, there are $a_i(t),b_i(t)\in\Omega_i:=\overline{\Omega}\cap[i,i+1]$ such that

$$\int_{\Omega_i}v(t,x)\mathrm{d}x\sim 1,\quad \int_{\Omega_i}\theta(t,x)\mathrm{d}x\sim 1,\quad v(t,a_i(t))\sim 1,\quad \theta(t,b_i(t))\sim 1. \tag{2.4}$$

We next derive a representation of the specific volume v for the general case $p\geq 1$ in order to obtain a positive lower bound for v.

Lemma 2.3
For all $(t,x)\in[0,T]\times\overline{\Omega}$, it holds that $v(t,x)\gtrsim \mathrm{e}^{-Ct}$.

Proof
Let $x\in\Omega_i$ for some fixed integer i. Because μ is a positive constant, we integrate $(1.1)_2$ over $[0,t]\times[a_i(t),x]$ to obtain

$$\int_{a_i(t)}^x[u(t,z)-u_0(z)]\,\mathrm{d}z+\int_0^t\left[\frac{\theta(s,x)}{v^p(s,x)}-\frac{\theta(s,a_i(t))}{v^p(s,a_i(t))}\right]\mathrm{d}s=\frac{\mu}{p}\ln\frac{v^p(t,x)v_0^p(a_i(t))}{v_0^p(x)v^p(t,a_i(t))},$$

which yields

$$\frac{1}{v^p(t,x)}\exp\left(\frac{p}{\mu}\int_0^t\frac{\theta(s,x)}{v^p(s,x)}\mathrm{d}s\right)=\frac{1}{B_i(t,x)Y_i(t)}, \tag{2.5}$$

where

$$B_i(t,x)=\frac{v_0^p(x)v^p(t,a_i(t))}{v_0^p(a_i(t))}\exp\left(\frac{p}{\mu}\int_{a_i(t)}^x(u(t,z)-u_0(z))\,\mathrm{d}z\right),$$
$$Y_i(t)=\exp\left(-\frac{p}{\mu}\int_0^t\frac{\theta(s,a_i(t))}{v^p(s,a_i(t))}\mathrm{d}s\right).$$

Multiplying (2.5) by $\frac{p}{\mu}\theta(t,x)$ gives

$$\frac{\partial}{\partial t}\exp\left(\frac{p}{\mu}\int_0^t\frac{\theta(s,x)}{v^p(s,x)}\mathrm{d}s\right)=\frac{p}{\mu}\frac{\theta(t,x)}{B_i(t,x)Y_i(t)},$$

which when combined with (2.5) yields

$$v(t,x)=\left[B_i(t,x)Y_i(t)+\frac{p}{\mu}\int_0^t\frac{B_i(t,x)Y_i(t)}{B_i(s,x)Y_i(s)}\theta(s,x)\mathrm{d}s\right]^{\frac{1}{p}}. \tag{2.6}$$

Because (2.1) implies

$$\left|\int_{a_i(t)}^x(u(t,z)-u_0(z))\,\mathrm{d}z\right|\leq\int_{\Omega_i}\left(u^2(t,z)+u_0^2(z)\right)\mathrm{d}z\lesssim 1,$$

we have from (2.4) that

$$B_i(t,x) \sim 1. \tag{2.7}$$

If we apply the Jensen inequality to the convex function z^p ($z > 0$) and use (2.4), we infer

$$\int_{\Omega_i} v^p(t,z)\mathrm{d}z \ge \left(\int_{\Omega_i} v(t,z)\mathrm{d}z\right)^p \gtrsim 1. \tag{2.8}$$

Hence,

$$Y_i^{-1}(t) \lesssim Y_i^{-1}(t)\int_{\Omega_i} v^p(t,z)\mathrm{d}z \lesssim 1 + \int_0^t Y_i^{-1}(s)\int_{\Omega_i}\theta(s,z)\mathrm{d}z\mathrm{d}s \lesssim 1 + \int_0^t Y_i^{-1}(s)\mathrm{d}s.$$

Apply the Gronwall inequality to derive $Y_i^{-1}(t) \lesssim \mathrm{e}^{Ct}$. Combining this last inequality with (2.6) and (2.7) completes the proof of this lemma. □

Next, we extend the argument in [4, 5] and derive a local representation of v for $p \ge 1$ to obtain positive lower and upper bounds on $v(t,x)$ independently of both t and x.

Lemma 2.4
For all $(t,z) \in [0,T] \times \overline{\Omega}$, it holds that $v(t,z) \sim 1$.

Proof
Let $z \in \overline{\Omega}$ be arbitrary but fixed. We divide the proof into three steps.

Step 1
Let $\varphi \in W^{1,\infty}(\mathbb{R})$ be defined by

$$\varphi(x) = \begin{cases} 1, & x < [z]+1, \\ [z]+2-x, & [z]+1 \le x < [z]+2, \\ 0, & x \ge [z]+2, \end{cases} \tag{2.9}$$

where $[x]$ denotes the largest integer that is less or equal to x.

Set $y \in I := ([z]-1,[z]+1) \cap \Omega$. Multiply $(1.1)_2$ by φ and integrate the resulting identity over (y,∞) to find

$$-\int_y^\infty (\varphi u)_t(t,x)\mathrm{d}x = \frac{\mu}{p}\left(\ln v^p\right)_t(t,y) - \frac{\theta}{v^p}(t,y) + \int_{[z]+1}^{[z]+2}\left(\mathcal{P} - \frac{\mu u_x}{v}\right)(t,x)\mathrm{d}x. \tag{2.10}$$

We integrate (2.10) over $[0,t]$ to deduce that

$$\frac{1}{v^p(t,y)}\exp\left(\frac{p}{\mu}\int_0^t \frac{\theta(s,y)}{v^p(s,y)}\mathrm{d}s\right) = \frac{1}{B(t,y)Y(t)} \tag{2.11}$$

with

$$B(t,y) = v_0^p(y)\exp\left(\frac{p}{\mu}\int_y^\infty \left(u_0(x)-u(t,x)\right)\varphi(x)\mathrm{d}x\right), \tag{2.12}$$

$$Y(t) = \exp\left(\frac{p}{\mu}\int_0^t\int_{[z]+1}^{[z]+2}\left(\frac{\mu u_x}{v} - \mathcal{P}\right)\mathrm{d}x\mathrm{d}s\right). \tag{2.13}$$

Multiply (2.11) by $p\theta(t,y)/\mu$ to obtain

$$\exp\left(\frac{p}{\mu}\int_0^t \frac{\theta(s,y)}{v^p(s,y)}\mathrm{d}s\right) = 1 + \frac{p}{\mu}\int_0^t \frac{\theta(s,y)}{B(s,y)Y(s)}\mathrm{d}s,$$

which when combined with (2.11) yields

$$v(t,y) = \left[B(t,y)Y(t) + \frac{p}{\mu}\int_0^t \frac{B(t,y)Y(t)}{B(s,y)Y(s)}\theta(s,y)\mathrm{d}s\right]^{\frac{1}{p}} \tag{2.14}$$

for $y \in I$, where B and Y are given by (2.12) and (2.13).

Step 2
Because

$$\left|\int_y^\infty \left(u_0(x)-u(t,x)\right)\varphi(x)\mathrm{d}x\right| \lesssim 1 + \int_y^{[z]+2}\left(u^2(t,x)+u_0^2(x)\right)\mathrm{d}x \lesssim 1,$$

we have

$$B(t,y)\sim 1. \tag{2.15}$$

Applying the Hölder and Young inequalities and the Jensen inequality to the convex function z^{-p} $(z>0)$, and utilizing (2.1) and (2.4), we discover

$$\begin{aligned}\int_\tau^t\int_{[z]+1}^{[z]+2}\left(\frac{\mu u_x}{v}-\mathcal{P}\right)&\le C(\epsilon)\int_\tau^t\int_{[z]+1}^{[z]+2}\frac{u_x^2}{v\theta}+\epsilon\int_\tau^t\int_{[z]+1}^{[z]+2}\frac{\theta}{v}-\int_\tau^t\int_{[z]+1}^{[z]+2}\frac{\theta}{v^p}\\&\le C(\epsilon)+\epsilon\int_\tau^t\int_{[z]+1}^{[z]+2}\theta-\frac{1}{2}\int_\tau^t\int_{[z]+1}^{[z]+2}\frac{\theta}{v^p}\\&\le C(\epsilon)+C\epsilon(t-\tau)-\frac{1}{2}\int_\tau^t\inf_{([z]+1,[z]+2)}\theta(\cdot,s)\left(\int_{[z]+1}^{[z]+2}v\right)^{-p}ds\\&\le C(\epsilon)+C\epsilon(t-\tau)-C^{-1}\int_\tau^t\inf_{([z]+1,[z]+2)}\theta(\cdot,s)ds\end{aligned} \tag{2.16}$$

for all $0<\epsilon\le\epsilon_0$ and some sufficiently small constant $\epsilon_0>0$. As shown in [4], the last term on the right-hand side of (2.16) can be controlled as

$$-\int_\tau^t\inf_{([z]+1,[z]+2)}\theta(\cdot,s)ds\le C-C^{-1}(t-\tau). \tag{2.17}$$

Plugging (2.17) into (2.16) and taking $\epsilon>0$ suitable small, we derive

$$\int_\tau^t\int_{[z]+1}^{[z]+2}\left(\frac{\mu u_x}{v}-\mathcal{P}\right)\le C-C^{-1}(t-\tau) \tag{2.18}$$

for $t\ge\tau\ge 0$. Recalling the definition (2.13) of $Y(t)$, we have from (2.18) that

$$0\le Y(t)\le Ce^{-t/C},\quad \frac{Y(t)}{Y(s)}\le Ce^{-(t-s)/C}\quad\text{for all } t\ge s\ge 0. \tag{2.19}$$

Step 3
Inserting (2.15) and (2.19) into (2.14), we obtain

$$v^p(t,y)\lesssim 1+\int_0^t e^{-(t-s)/C}\theta(s,y)ds. \tag{2.20}$$

In view of (2.4), we apply the Hölder inequality to obtain

$$\left|\theta^{\frac12}(t,y)-\theta^{\frac12}(t,b_{[z]}(t))\right|\lesssim\int_I\theta^{-\frac12}|\theta_x|(t,x)dx\lesssim\sup_I v^{\frac12}(t,\cdot)\left[\int_I\frac{\theta_x^2}{v\theta^2}dx\right]^{\frac12},$$

which along with (2.4) implies

$$1-C\sup_I v(t,\cdot)\int_I\frac{\theta_x^2}{v\theta^2}dx\lesssim\theta(t,y)\lesssim 1+\sup_I v(t,\cdot)\int_I\frac{\theta_x^2}{v\theta^2}dx\quad\text{for all } y\in I. \tag{2.21}$$

Plug the second inequality in (2.21) into (2.20) to

$$\sup_I v^p(t,\cdot)\lesssim 1+\int_0^t\sup_I v(s,\cdot)\int_I\frac{\theta_x^2}{v\theta^2}(s,x)dxds.$$

In light of (2.1), we apply the Young and Gronwall inequalities to this last estimate to derive

$$\sup_I v(t,\cdot)\lesssim 1\quad\text{for all } t\in[0,T]. \tag{2.22}$$

It follows from (2.4), (2.8), (2.14), (2.15), and (2.19) that

$$1\lesssim\int_I v^p(t,y)dy\lesssim\int_I Y(t)dy+\int_0^t\frac{Y(t)}{Y(s)}\int_I\theta(s,y)dyds\lesssim e^{-t/C}+\int_0^t\frac{Y(t)}{Y(s)}ds.$$

Consequently, we have

$$\int_0^t \frac{Y(t)}{Y(s)} \mathrm{d}s \gtrsim 1 - C\mathrm{e}^{-t/C}. \tag{2.23}$$

Then we have from (2.1), (2.14), and (2.19)–(2.23) that

$$\begin{aligned} v^p(t,y) &\gtrsim \int_0^t \frac{Y(t)}{Y(s)} \left(1 - C\int_I \frac{\theta_x^2}{v\theta^2}\mathrm{d}x\right)\mathrm{d}s \\ &\gtrsim 1 - C_1\mathrm{e}^{-t/C_2} - C\left(\int_0^{t/2} + \int_{t/2}^t\right)\frac{Y(t)}{Y(s)}\int_I \frac{\theta_x^2}{v\theta^2}\mathrm{d}x\mathrm{d}s \\ &\gtrsim 1 - C_1\mathrm{e}^{-t/C_2} - C\int_0^{t/2} \mathrm{e}^{-(t-s)/C}\int_I \frac{\theta_x^2}{v\theta^2}\mathrm{d}x\mathrm{d}s - C\int_{t/2}^t\int_I \frac{\theta_x^2}{v\theta^2}\mathrm{d}x\mathrm{d}s \\ &\gtrsim 1 - C_1\mathrm{e}^{-t/C_2} - C\mathrm{e}^{-t/(2C)} - C\int_{t/2}^t\int_I \frac{\theta_x^2}{v\theta^2}\mathrm{d}x\mathrm{d}s \\ &\gtrsim 1 \quad \text{for all } y \in I,\ t \geq T_0, \end{aligned} \tag{2.24}$$

where T_0 is a positive constant independent of t. In particular, the estimate (2.24) gives us

$$v(t,z) \gtrsim 1 \quad \text{for all } z \in \Omega,\ t \geq T_0. \tag{2.25}$$

Therefore, we can complete the proof of this lemma by combining (2.22), (2.25), and Lemma 2.3.

□

We have thus proved that $v(x,t)$ is bounded from below and above independently of both t and x. From this point on, all the remaining arguments that are needed to deduce a priori estimates are standard, which have been discussed in [8, 9]. Theorem 1 then follows from the standard continuation argument.

Acknowledgements

This work was supported by the 'Fundamental Research Funds for the Central Universities'. The authors express much gratitude to Professor Huijiang Zhao for his support and advice.

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