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Topology identification of weighted complex dynamical networks

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Abstract

Recently, various papers investigated the geometry features, synchronization and control of complex network provided with certain topology. While, the exact topology of a network is sometimes unknown or uncertain. Using Lyapunov theory, we propose an adaptive feedback controlling method to identify the exact topology of a rather general weighted complex dynamical network model. By receiving the network nodes evolution, the topology of such kind of network with identical or different nodes, or even with switching topology can be monitored. Experiments show that the methods presented in this paper are of high accuracy with good performance.

Keywords: Complex network; Weight; Topology identification; Variable tracing

1. Introduction

Nowadays, complex networks exist in every corner of the world, from communication networks to social networks, from cellular networks to metabolic networks, from Internet to World Wide Web [1–11]. Research on complex networks is already in the ascendant.

Many existing literatures focused on investigating the geometry features, synchronization and control of complex dynamical network provided with certain topology [12–20]. While, in the real world, the exact topology of a complex dynamical network is sometimes unknown or uncertain [21]. In view of its characteristics, identifying topology of complex network becomes a key problem in many disciplines. Such as the protein–DNA (deoxyribonucleic acid) interactions in the regulation of various cellular processes [1,6–8]. Protein–DNA interactions play pivotal roles in many cell processes, such as DNA replication, modification, repair and RNA (ribonucleic acid) transcription. It is of significance that by monitoring dynamic behavior of proteins during the process of recognition through NMR (nuclear magnetic resonance) technology [8], the topology structure of the interaction network, though in which different kinds of nodes may exist, can be identified. Another interesting case is the stock network. In the stock market, one of the main reasons responsible for the fluctuations of it—the price changes per unit time—is believed to be herding behavior,

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which is characterized by the fact that decision-making of single trader is influenced by that of others [9,10]. But how much effect on them by each of other people? Maybe the investors themselves have no clear idea. While through monitoring the behavior of all the members influenced in a community, the interactions among them can be investigated and quantified systematically. Except what have been mentioned above, this technology can be implemented to many other fields, such as remote control and diagnostics, disease transmission, management and administration of cyber bar, and so on.

Therefore, topology identification becomes critical in the research of complex networks. By adaptive feedback controlling method, the real network is served as a drive network, and we construct another response network receiving the evolution of each node, then the exact topology of the real network can be identified. Along with it, the evolution of every node is traced. Using Lyapunov stability theory [22], mathematical analysis of the mechanism is developed rigorously. Our controlling approach can be applied to a large amount of rather general weighted complex dynamical networks not only with identical nodes, but also with different nodes. Besides, even when the topology of the complex dynamical network changes, it can be monitored as well. All these will contribute to improving efficiency and accuracy of network analysis.

The left paper is organized as follows. Section 2 describes the topology identification method for a general weighted complex dynamical network with identical nodes. Identifying topology mechanism for such kind of network consisting of different nodes are detailed in Section 3. Section 4 gives three computational examples include network with identical nodes, network with different nodes and switching network with different nodes to illustrate effectiveness of the proposed approach. The main ideas and conclusions are summarized up in Section 5.

2. A weighted complex network with identical node dynamics

2.1. Model and assumption

Consider a weighted complex dynamical network consisting of N identical nodes with linearly couplings, which is characterized by

$$\dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i) + \sum_{j=1}^{N} c_{ij}\mathbf{A}\mathbf{x}_j, \tag{1}$$

where $1 \leqslant i \leqslant N$, $\mathbf{x}_i = (x_{i1}, x_{i2}, \ldots, x_{in})^{\mathrm{T}} \in \mathbf{R}^n$ is the state vector of the ith node, $\mathbf{f} : \mathbf{R}^n \times R^+ \to \mathbf{R}^n$ is a smooth nonlinear vector field, node dynamics is $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$, $\mathbf{A} \in \mathbf{R}^{n\times n}$ is the inner-coupling matrix and $\mathbf{C} = (c_{ij})_{N\times N} \in \mathbf{R}^{N\times N}$ is the unknown or uncertain weight configuration matrix. If there is a link from node i to node j $(j \neq i)$, then $c_{ij} \neq 0$ and c_{ij} is the weight; otherwise, $c_{ij} = 0$.

In this model, the inner coupling matrix $\mathbf{A}$ is not necessarily symmetric, and the weight configuration matrix $\mathbf{C}$ needs not to be symmetric, irreducible and diffusive. For the purpose of identifying these unknown or uncertain weights, namely for identifying topology of the model, a useful assumption is introduced.

Assumption 1 (*A1*). Suppose that there exists a positive constant α satisfying

$$\|\mathbf{f}(\mathbf{y}) - \mathbf{f}(\mathbf{z})\| \leqslant \alpha \|\mathbf{y} - \mathbf{z}\|,$$

where $\mathbf{y}$, $\mathbf{z}$ are time-varying vectors, $\|\cdot\|$ represents 2-norm.

2.2. Adaptive controlling method

We give some insights into topology identification of model (1). Consider another generally controlled complex dynamical network as follows:

$$\dot{\hat{\mathbf{x}}}_i = \mathbf{f}(\hat{\mathbf{x}}_i) + \sum_{j=1}^{N} \hat{c}_{ij}\mathbf{A}\hat{\mathbf{x}}_j + \mathbf{u}_i, \tag{2}$$

where $1 \leqslant i \leqslant N$, $\hat{\mathbf{x}}_i = (\hat{x}_{i1}, \hat{x}_{i2}, \ldots, \hat{x}_{in})^{\mathrm{T}} \in \mathbf{R}^n$ is the response state vector of the ith node, $\mathbf{u}_i \in \mathbf{R}^n$ is the control input, and $\hat{c}_{ij}$ is the estimation of the weight c_{ij}.

Assume that $\tilde{\mathbf{x}}_i = \hat{\mathbf{x}}_i - \mathbf{x}_i$ and $\tilde{c}_{ij} = \hat{c}_{ij} - c_{ij}$, we have the error system:

$$\dot{\tilde{\mathbf{x}}}_i = \mathbf{f}(\hat{\mathbf{x}}_i) - \mathbf{f}(\mathbf{x}_i) + \sum_{j=1}^{N} \tilde{c}_{ij}\mathbf{A}\hat{\mathbf{x}}_j + \sum_{j=1}^{N} c_{ij}\mathbf{A}\tilde{\mathbf{x}}_j + \mathbf{u}_i, \tag{3}$$

where $1 \leqslant i \leqslant N$.

On the assumption of A1, the following theorem can be deduced.

Theorem 1. *Suppose that* A1 *holds. The weight configuration matrix* $\mathbf{C}$ *of general linearly coupled complex dynamical network* (1) *can be identified by the estimation* $\hat{\mathbf{C}}$ *using the following response network*:

$$\begin{cases} \dot{\hat{\mathbf{x}}}_i = \mathbf{f}(\hat{\mathbf{x}}_i) + \sum\limits_{j=1}^{N} \hat{c}_{ij}\mathbf{A}\hat{\mathbf{x}}_j + \mathbf{u}_i, \\ \dot{\hat{c}}_{ij} = -\tilde{\mathbf{x}}_i^{\mathrm{T}}\mathbf{A}\hat{\mathbf{x}}_j, \\ \mathbf{u}_i = -d_i\tilde{\mathbf{x}}_i, \\ \dot{d}_i = k_i\|\tilde{\mathbf{x}}_i\|^2, \end{cases} \tag{4}$$

where $1 \leqslant i,\ j \leqslant N$, $1 \leqslant i \leqslant N$ *is any positive constant.*

Proof. Since A1 holds, we have $\|\mathbf{f}(\hat{\mathbf{x}}_i, t) - \mathbf{f}(\mathbf{x}_i, t)\| \leqslant \alpha\|\tilde{\mathbf{x}}_i\|$, where $1 \leqslant i \leqslant N$. Then choose Lyapunov candidate as

$$V = \frac{1}{2}\sum_{i=1}^{N} \tilde{\mathbf{x}}_i^{\mathrm{T}}\tilde{\mathbf{x}}_i + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \tilde{c}_{ij}^2 + \frac{1}{2}\sum_{i=1}^{N} \frac{1}{k_i}(d_i - d^*)^2,$$

where d^* is a sufficiently large positive constant to be determined, we have the differential coefficient of V as

$$\begin{aligned} \dot{V} = & \sum_{i=1}^{N} \tilde{\mathbf{x}}_i^{\mathrm{T}}\dot{\tilde{\mathbf{x}}}_i + \sum_{i=1}^{N}\sum_{j=1}^{N} \tilde{c}_{ij}\dot{\hat{c}}_{ij} + \sum_{i=1}^{N} \frac{1}{k_i}(d_i - d^*)\dot{d}_i \\ \leqslant & \sum_{i=1}^{N} \alpha\|\tilde{\mathbf{x}}_i\|^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} \tilde{c}_{ij}\tilde{\mathbf{x}}_i^{\mathrm{T}}\mathbf{A}\hat{\mathbf{x}}_j + \sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij}\tilde{\mathbf{x}}_i^{\mathrm{T}}\mathbf{A}\tilde{\mathbf{x}}_j \\ & - \sum_{i=1}^{N} d_i\|\tilde{\mathbf{x}}_i\|^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} \tilde{c}_{ij}\dot{\hat{c}}_{ij} + \sum_{i=1}^{N}(d_i - d^*)\|\tilde{\mathbf{x}}_i\|^2 \\ = & \sum_{i=1}^{N}(\alpha - d^*)\|\tilde{\mathbf{x}}_i\|^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij}\tilde{\mathbf{x}}_i^{\mathrm{T}}\mathbf{A}\tilde{\mathbf{x}}_j \\ = & \tilde{\mathbf{X}}^{\mathrm{T}}\mathbf{P}\tilde{\mathbf{X}}, \end{aligned}$$

where $\tilde{\mathbf{X}} = (\tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2, \ldots, \tilde{\mathbf{x}}_N)^{\mathrm{T}}$, $\mathbf{P} = (\alpha - d^*)\mathbf{I}_{nN} + \mathbf{C} \otimes \mathbf{A}$, and $\mathbf{I}_{nN}$ represents identity matrix with $n \times N$ dimension. It is obvious that the matrix $\mathbf{P}$ is negative definite for sufficiently large positive constants d^*. So, the set $\mathbf{M} = \{\tilde{\mathbf{x}}_i = 0, \tilde{c}_{ij} = 0, d_i = d^*, \quad i, j = 1, 2, \ldots, N\}$ is the largest invariant set of the set $\mathbf{M}' = \{\dot{V} = 0\}$ for the error system (3). According to the invariant principle of functional differential equation [22,26], starting with arbitrary initial values, the trajectory asymptotically converges to the set $\mathbf{M}$. Therefore, one gets $\lim_{t\to+\infty} \tilde{\mathbf{x}}_i = \mathbf{0}$ for $1 \leqslant i \leqslant N$ and $\lim_{t\to+\infty} \tilde{c}_{ij} = 0$ for $1 \leqslant i,\ j \leqslant N$. As a result, the weight configuration matrix $\mathbf{C}$ can be identified by the matrix $\hat{\mathbf{C}}$. Thus complete the proof. □

According to Theorem 1, we get the idea that using our adaptive feedback controlling method, the weights of the complex dynamical network (1) can be estimated by receiving the evolution of network nodes. Namely, the exact topology of model (1) can be identified. In addition, we find that the evolution of every node is traced along with it.

3. A weighted complex network with different node dynamics

3.1. Model and assumption

In this subsection, we consider a weighted complex dynamical network consisting of different node dynamics which is described by

$$\begin{cases} \dot{\mathbf{x}}_i = \mathbf{g}(\mathbf{x}_i) + \sum_{j=1}^{N} c_{ij}\mathbf{A}\mathbf{x}_j, & 1 \leqslant i \leqslant N^*, \\ \dot{\mathbf{x}}_i = \mathbf{h}(\mathbf{x}_i) + \sum_{j=1}^{N} c_{ij}\mathbf{A}\mathbf{x}_j, & N^*+1 \leqslant i \leqslant N, \end{cases} \tag{5}$$

where $\mathbf{g}, \mathbf{h} : \mathbf{R}^n \times R^+ \to \mathbf{R}^n$ are different smooth nonlinear vector functions.

Similarly, a useful hypothesis is given as follows:

Assumption 2 (*A2*). Suppose that there exist positive constants β and γ, satisfying

$$\|\mathbf{g}(\mathbf{y}) - \mathbf{g}(\mathbf{z})\| \leqslant \beta\|\mathbf{y} - \mathbf{z}\|, \quad \|\mathbf{h}(\mathbf{y}) - \mathbf{h}(\mathbf{z})\| \leqslant \gamma\|\mathbf{y} - \mathbf{z}\|,$$

where $\mathbf{y}$, $\mathbf{z}$ are time-varying vectors.

3.2. Adaptive controlling method

For the sake of identifying topology of network model (5) and tracing network nodes evolution, another generally controlled complex dynamical network is introduced here:

$$\begin{cases} \dot{\hat{\mathbf{x}}}_i = \mathbf{g}(\hat{\mathbf{x}}_i) + \sum_{j=1}^{N} \hat{c}_{ij}\mathbf{A}\hat{\mathbf{x}}_j + \mathbf{u}_i, & 1 \leqslant i \leqslant N^*, \\ \dot{\hat{\mathbf{x}}}_i = \mathbf{h}(\hat{\mathbf{x}}_i) + \sum_{j=1}^{N} \hat{c}_{ij}\mathbf{A}\hat{\mathbf{x}}_j + \mathbf{u}_i, & N^*+1 \leqslant i \leqslant N. \end{cases} \tag{6}$$

Then, we have the error system:

$$\begin{cases} \dot{\tilde{\mathbf{x}}}_i = \mathbf{g}(\hat{\mathbf{x}}_i) - \mathbf{g}(\mathbf{x}_i) + \sum_{j=1}^{N} \tilde{c}_{ij}\mathbf{A}\hat{\mathbf{x}}_j + \sum_{j=1}^{N} c_{ij}\mathbf{A}\tilde{\mathbf{x}}_j + \mathbf{u}_i, & 1 \leqslant i \leqslant N^*, \\ \dot{\tilde{\mathbf{x}}}_i = \mathbf{h}(\hat{\mathbf{x}}_i) - \mathbf{h}(\mathbf{x}_i) + \sum_{j=1}^{N} \tilde{c}_{ij}\mathbf{A}\hat{\mathbf{x}}_j + \sum_{j=1}^{N} c_{ij}\mathbf{A}\tilde{\mathbf{x}}_j + \mathbf{u}_i, & N^*+1 \leqslant i \leqslant N. \end{cases} \tag{7}$$

Similarly, the following adaptive controlling mechanism can be deduced:

Theorem 2. *Suppose that* A2 *holds. The weight configuration matrix* $\mathbf{C}$ *of general linearly coupled complex dynamical network* (5) *can be identified by the estimation* $\hat{\mathbf{C}}$ *using the following response network*:

$$\begin{cases} \dot{\hat{\mathbf{x}}}_i = \mathbf{g}(\hat{\mathbf{x}}_i) + \sum_{j=1}^{N} \hat{c}_{ij}\mathbf{A}\hat{\mathbf{x}}_j + \mathbf{u}_i, & 1 \leqslant i \leqslant N^*, \\ \dot{\hat{\mathbf{x}}}_i = \mathbf{h}(\hat{\mathbf{x}}_i) + \sum_{j=1}^{N} \hat{c}_{ij}\mathbf{A}\hat{\mathbf{x}}_j + \mathbf{u}_i, & N^*+1 \leqslant i \leqslant N, \\ \dot{\hat{\mathbf{c}}}_{ij} = -\tilde{\mathbf{x}}_i^{\mathrm{T}}\mathbf{A}\hat{\mathbf{x}}_j, & 1 \leqslant i,\ j \leqslant N, \\ \mathbf{u}_i = -e_i\tilde{\mathbf{x}}_i, & \\ \dot{e}_i = l_i\|\tilde{\mathbf{x}}_i\|^2, & 1 \leqslant i \leqslant N, \end{cases} \tag{8}$$

where l_i $(1 \leqslant i \leqslant N)$ *is any positive constant.*

Proof. Provided with the condition A2, we get $\|\mathbf{g}(\hat{\mathbf{x}}_i,t)-\mathbf{g}(\mathbf{x}_i,t)\|\leqslant\beta\|\tilde{\mathbf{x}}_i\|$ for $1\leqslant i\leqslant N^*$ and $\|\mathbf{h}(\hat{\mathbf{x}}_i,t)-\mathbf{h}(\mathbf{x}_i,t)\|\leqslant\gamma\|\tilde{\mathbf{x}}_i\|$ for $N^*+1\leqslant i\leqslant N$. Choose Lyapunov function as

$$V=\frac{1}{2}\sum_{i=1}^{N}\tilde{\mathbf{x}}_i^{\mathrm{T}}\tilde{\mathbf{x}}_i+\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}^2+\frac{1}{2}\sum_{i=1}^{N}\frac{1}{l_i}(e_i-e^*)^2,$$

where e^* is a sufficiently large positive constant to be determined. We then have

$$\begin{aligned}\dot{V}&=\sum_{i=1}^{N^*}\tilde{\mathbf{x}}_i^{\mathrm{T}}\dot{\tilde{\mathbf{x}}}_i+\sum_{i=N^*+1}^{N}\tilde{\mathbf{x}}_i^{\mathrm{T}}\dot{\tilde{\mathbf{x}}}_i+\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}\dot{\hat{c}}_{ij}+\sum_{i=1}^{N}\frac{1}{l_i}(e_i-e^*)\dot{e}_i\\&\leqslant\sum_{i=1}^{N^*}\beta\|\tilde{\mathbf{x}}_i\|^2+\sum_{i=N^*+1}^{N}\gamma\|\tilde{\mathbf{x}}_i\|^2+\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}\tilde{\mathbf{x}}_i^{\mathrm{T}}\mathbf{A}\hat{\mathbf{x}}_j+\sum_{i=1}^{N}\sum_{j=1}^{N}c_{ij}\tilde{\mathbf{x}}_i^{\mathrm{T}}\mathbf{A}\tilde{\mathbf{x}}_j\\&\quad-\sum_{i=1}^{N}e_i\|\tilde{\mathbf{x}}_i\|^2+\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}\dot{\hat{c}}_{ij}+\sum_{i=1}^{N}(e_i-e^*)\|\tilde{\mathbf{x}}_i\|^2\\&=\tilde{\mathbf{X}}^{\mathrm{T}}\mathbf{Q}\tilde{\mathbf{X}},\end{aligned}$$

where

$$\mathbf{Q}=\begin{pmatrix}(\beta-e^*)\mathbf{I}_{nN^*}&\mathbf{0}\\\mathbf{0}&(\gamma-e^*)\mathbf{I}_{n(N-N^*)}\end{pmatrix}+\mathbf{C}\otimes\mathbf{A}.$$

Clearly, the matrix $\mathbf{Q}$ is negative definite when the positive constant e^* is large enough. Similar to the proof method of Theorem 1, we obtain $\lim_{t\to+\infty}\tilde{\mathbf{x}}_i=\mathbf{0}$ for $1\leqslant i\leqslant N$ and $\lim_{t\to+\infty}\tilde{c}_{ij}=0$ for $1\leqslant i,\ j\leqslant N$. That is, the weight configuration matrix $\mathbf{C}$ can be identified by the matrix $\hat{\mathbf{C}}$. Thus the proof is completed.

Remark 1. In this theorem, the weighted complex dynamical network is built up of two types of different nodes. For networks with more types of ones, similar work can be generalized easily.

From this theorem, it is shown that using similar adaptive feedback controlling approach, the exact topology of model (5) can be identified, and the evolution of every node can be traced at the same time. On account of the widespread circumstances in which considerable weighted complex dynamical networks with different nodes exist, this mechanism is of great significance in practice.

4. Numerical simulation

4.1. A weighted complex network with identical node dynamics

In this subsection, a simple example is used to show the effectiveness of the adaptive controlling method presented in Section 1.

It is well known that Lorenz, Chen, Lü systems are several typical benchmark chaotic systems [23–25]. In which, Lorenz chaotic system is known as

$$\begin{aligned}\dot{\mathbf{x}}&=\begin{pmatrix}-a&a&0\\c&-1&0\\0&0&-b\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}+\begin{pmatrix}0\\-x_1x_3\\x_1x_2\end{pmatrix}\\&\triangleq\mathbf{B}_1\mathbf{x}+\mathbf{W}(\mathbf{x}),\end{aligned}$$

when $a=10, b=\frac{8}{3}, c=28$, and Lü chaotic system is known as

$$\begin{aligned}\dot{\mathbf{x}} &= \begin{pmatrix} -a & a & 0 \\ 0 & c & 0 \\ 0 & 0 & -b \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ -x_1x_3 \\ x_1x_2 \end{pmatrix} \\ &\triangleq \mathbf{B}_2\mathbf{x} + \mathbf{W}(\mathbf{x}),\end{aligned}$$

when $a=36, b=3, c=20$. For any two state vectors $\mathbf{y}$ and $\mathbf{z}$ of Lorenz system or Lü system, there exists a constant M satisfying $\|y_p\|, \|z_p\| \leqslant M$ for $1 \leqslant p \leqslant 3$ since Lorenz attractor and Lü attractor are bounded by certain region, respectively. Therefore, one has

$$\begin{aligned}\|\mathbf{W}(\mathbf{y}) - \mathbf{W}(\mathbf{z})\| &\leqslant \sqrt{(-y_3(y_1-z_1) - z_1(y_3-z_3))^2 + (y_2(y_1-z_1) + z_1(y_2-z_2))^2} \\ &\leqslant 2M\|\mathbf{y}-\mathbf{z}\|.\end{aligned}$$

Thus Lorenz system and Lü system satisfy Assumption A1.

Here, we consider a weighted linearly coupled complex dynamical network (1) consists of 4 identical Lü systems, which is described by Fig. 1. This model can be written as

$$\dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i) + \sum_{j=1}^{4} c_{ij}\mathbf{A}\mathbf{x}_j, \tag{9}$$

where $1 \leqslant i \leqslant 4$, $\mathbf{f}(\mathbf{x}) = \mathbf{B}_2\mathbf{x} + \mathbf{W}(\mathbf{x})$, $\mathbf{C} = (c_{ij})_{4\times 4}$ is the weight configuration matrix with

$$\mathbf{C} = \begin{pmatrix} -6 & 4 & 2 & 0 \\ 4 & -6 & 3 & -1 \\ 2 & 3 & -5 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}.$$

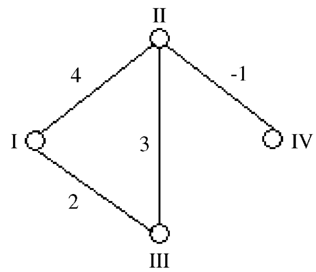

Fig. 1. Topology of model (9).

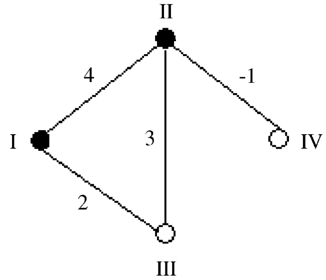

Fig. 2. Topology of model (11).

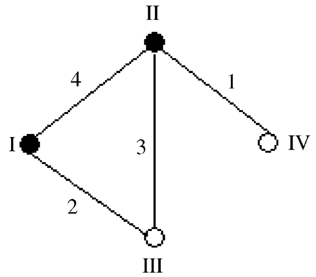

Fig. 3. Topology after switching.

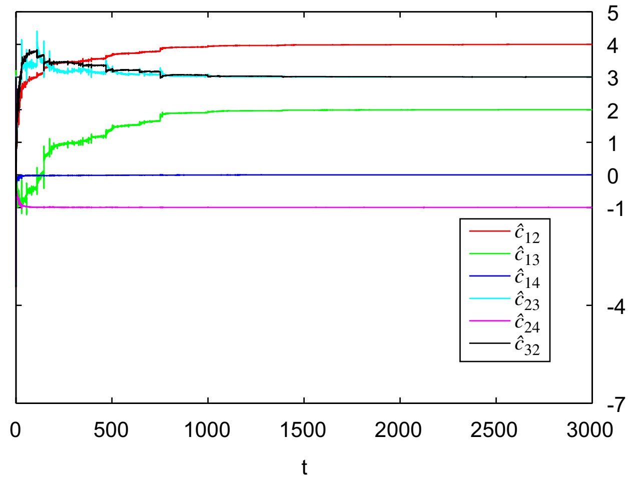

Fig. 4. Estimation of topology of model (9).

For simplicity, we assume that $\mathbf{A} = \mathbf{I}_3$. Since A1 holds, according to Theorem 1, the weigh configuration matrix $\mathbf{C}$ can be estimated by the matrix $\hat{\mathbf{C}}$ using the following controlled response network

$$\begin{cases} \dot{\hat{\mathbf{x}}}_i = \mathbf{f}(\hat{\mathbf{x}}_i) + \sum_{j=1}^{4} \hat{c}_{ij}\mathbf{A}\hat{\mathbf{x}}_j - d_i\tilde{\mathbf{x}}_i, \\ \dot{\hat{\mathbf{c}}}_{ij} = -\tilde{\mathbf{x}}_i^{\mathrm{T}}\mathbf{A}\hat{\mathbf{x}}_j, \\ \dot{d}_i = k_i\|\tilde{\mathbf{x}}_i\|^2, \end{cases} \tag{10}$$

where $1 \leqslant i \leqslant 4$. Some elements of matrix $\hat{\mathbf{C}}$ are displayed in Fig. 4 and the synchronous errors $\tilde{\mathbf{x}}_i$ $(1 \leqslant i \leqslant 4)$ are shown in Fig. 5.

In the numerical simulation, all parameters are given as follows: $k_i = 1, d_i(0) = 1$, $\mathbf{x}_i(0) = (3.5 + 0.5i, 4.5 + 0.5i, 5.5 + 0.5i)^{\mathrm{T}}$, $\hat{\mathbf{x}}_i(0) = (5.5 + 0.5i, 6.5 + 0.5i, 7.5 + 0.5i)^{\mathrm{T}}$, $\hat{c}_{ij}(0) = 1$, where $1 \leqslant i,\ j \leqslant 4$.

4.2. A weighted complex network with different node dynamics

Next, we consider a weighted complex dynamical network (5) consists of four different node systems with two Lorenz systems (I, II node) and two Lü systems (III, IV node), which is characterized by Fig. 2. The model can be written as

$$\begin{cases} \dot{\mathbf{x}}_i = \mathbf{g}(\mathbf{x}_i) + \sum_{j=1}^{4} c_{ij}\mathbf{A}\mathbf{x}_j, & 1 \leqslant i \leqslant 2, \\ \dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i) + \sum_{j=1}^{4} c_{ij}\mathbf{A}\mathbf{x}_j, & 3 \leqslant i \leqslant 4, \end{cases} \tag{11}$$

where $\mathbf{g}(\mathbf{x}) = \mathbf{B}_1\mathbf{x} + \mathbf{W}(\mathbf{x})$, $\mathbf{A}$ and $\mathbf{C}$ are the same as that in model (9).

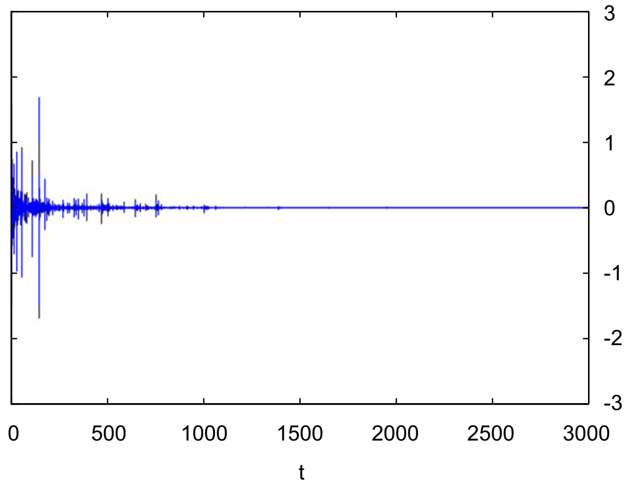

Fig. 5. Errors of model (9) ($\tilde{\mathbf{x}}_{ij}(1\leqslant i\leqslant 4, 1\leqslant j\leqslant 3)$).

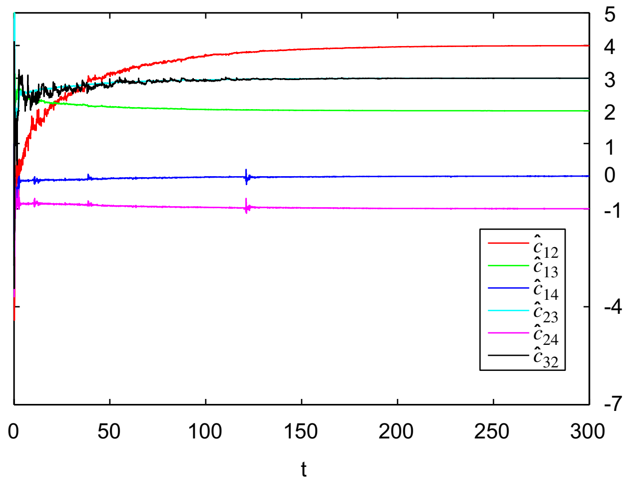

Fig. 6. Estimation of topology of model (11).

It is clear that A2 holds. Then according to Theorem 2, the weight configuration matrix $\mathbf{C}$ can be traced by the matrix $\hat{\mathbf{C}}$ using the controlled response network

$$\begin{cases} \dot{\hat{\mathbf{x}}}_i = \mathbf{g}(\hat{\mathbf{x}}_i) + \sum_{j=1}^{4} \hat{c}_{ij}\mathbf{A}\hat{\mathbf{x}}_j - e_i\tilde{\mathbf{x}}_i, & 1\leqslant i\leqslant 2, \\ \dot{\hat{\mathbf{x}}}_i = \mathbf{f}(\hat{\mathbf{x}}_i) + \sum_{j=1}^{4} \hat{c}_{ij}\mathbf{A}\hat{\mathbf{x}}_j - e_i\tilde{\mathbf{x}}_i, & 3\leqslant i\leqslant 4, \\ \dot{\hat{\mathbf{c}}}_{ij} = -\tilde{\mathbf{x}}_i^{\mathrm{T}}\mathbf{A}\hat{\mathbf{x}}_j, & 1\leqslant i,\ j\leqslant 4, \\ \dot{e}_i = l_i\|\tilde{\mathbf{x}}_i\|^2, & 1\leqslant i\leqslant 4. \end{cases} \tag{12}$$

Some elements of matrix $\hat{\mathbf{C}}$ are shown in Fig. 6 and the synchronous errors $\tilde{\mathbf{x}}_i$ $(1\leqslant i\leqslant 4)$ are displayed in Fig. 7.

All the parameters in this simulation are: $l_i = 1, e_i(0) = 1$, $\mathbf{x}_i(0) = (3.5 + 0.5i, 4.5 + 0.5i, 5.5 + 0.5i)^{\mathrm{T}}$, $\hat{\mathbf{x}}_i(0) = (5.5 + 0.5i, 6.5 + 0.5i, 7.5 + 0.5i)^{\mathrm{T}}$, $\hat{c}_{ij}(0) = 1$, where $1\leqslant i,\ j\leqslant 4$.

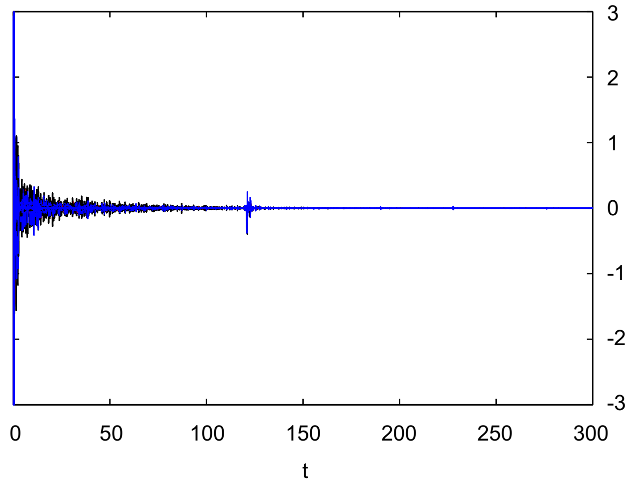

Fig. 7. Errors of model (11) ($\tilde{\mathbf{x}}_{ij}(1 \leqslant i \leqslant 4, 1 \leqslant j \leqslant 3)$).

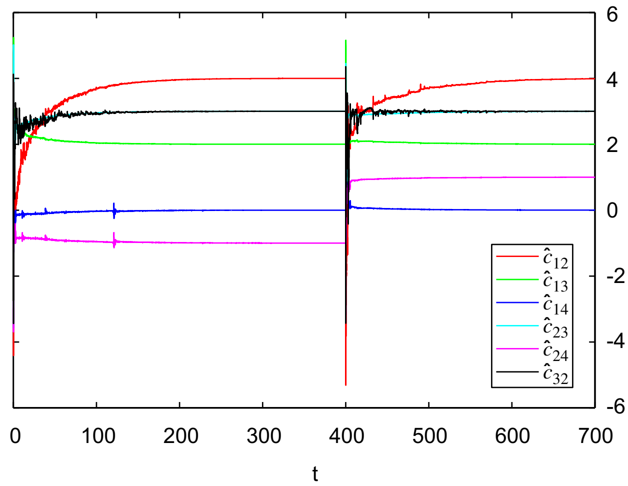

Fig. 8. Estimation of topology of the switching model.

4.3. A switching weighted complex network with different node dynamics

Then, we give an example of the switching weighted complex dynamical network which is characterized by a modified model of (11) with switching weight configuration **C**. At first, the weight structure of this model is described by Fig. 2; while at $t = 400$, the weight between nodes II and IV is changed from -1 to 1, thus the topology can be transformed into what has been plotted in Fig. 3. That is, the elements c_{24}, c_{22} and c_{44} are changed into 1, 1, -8 and -1, respectively.

From Figs. 8 and 9, it can be seen that the change of topology and the state variables have been traced effectively.

5. Conclusions

In this paper, we have presented an adaptive feedback controlling method to identify the exact topology of weighted complex dynamical network using Lyapunov stability theory. Particularly, the weight configuration is not necessarily symmetric, irreducible and diffusive, and the inner coupling matrix needs not to be

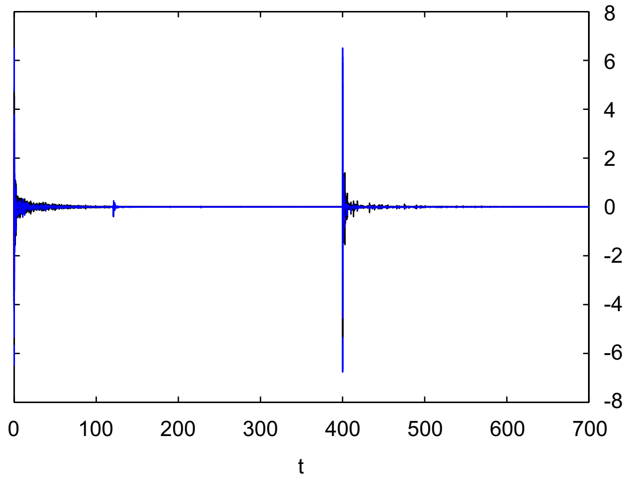

Fig. 9. Errors of the switching model ($\tilde{\mathbf{x}}_{ij}(1 \leqslant i \leqslant 4, 1 \leqslant j \leqslant 3)$).

symmetric. Two rigorous theorems have been deduced by mathematical analysis in Sections 2 and 3. Three computational examples of weighted complex dynamical network with identical nodes, different nodes and switching topology have been shown to illustrate the effectiveness of the proposed approach.

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